AdS Taub-NUT space and the $O(N)$ vector model on a squashed 3-sphere

Mark D.Yonge

Department of Physics, University of Wales Swansea,
Singleton Park, Swansea, SA2 8PP, U.K.
E-mail: `370058@swan.ac.uk`

ABSTRACT: In this note, motivated by the Klebanov-Polyakov conjecture we investigate the strongly coupled $O(N)$ vector model at large N on a squashed three-sphere and its holographic relation to bulk gravity on asymptotically locally AdS_4 spaces. We present analytical results for the action of the field theory as the squashing parameter $\alpha \to -1$, when the boundary becomes effectively one dimensional. The dual bulk geometry is AdS-Taub-NUT space in the corresponding limit. In this limit we solve the theory exactly and show that the action of the strongly coupled boundary theory scales as $\ln(1+\alpha)/(1+\alpha)^2$. This result is remarkably close to the $-1/(1+\alpha)^2$ scaling of the Einstein gravity action for AdS-Taub-NUT space. These results explain the numerical agreement presented in hep-th/0503238, and the soft logarithmic departure is interpreted as a prediction for the contribution due to higher spin fields in the bulk AdS_4 geometry.

KEYWORDS: Field Theories in Lower Dimensions, AdS-CFT Correspondence.

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Contents

1. Introduction and summary

The AdS/CFT correspondence states that string theories in asymptotically AdS spacetimes with d dimensions are dual to certain conformal field theories in $d - 1$ dimensions [1–3]. Testing these dualities is in general difficult because the theories involved are very complicated and are only tractable in different limiting regions of parameter space. However in [4] Klebanov and Polyakov suggested that a simpler duality exists between the large N limit of the singlet sector of the critical $O(N)$ vector model in three dimensions and the minimal bosonic higher spin gauge theory in four dimensional Anti de Sitter space. In [5] an extension of this duality was proposed between the $O(N)$ model on a squashed three sphere and the higher spin gauge theory on AdS Taub-NUT and AdS Taub-Bolt geometries with a phase transition occurring between the two on the gravitational side. The squashed three sphere is an S^1 bundle over S^2 with metric

$$ds^2 = \frac{a^2}{4}\left(\sigma_1^2 + \sigma_2^2 + \frac{\sigma_3^2}{1+\alpha}\right). \tag{1.1}$$

Where the σ_i are defined by:

$$\sigma_1 + i\sigma_2 = e^{-i\psi}(d\theta + i\sin\theta d\phi) \tag{1.2}$$

and

$$\sigma_3 = d\phi + \cos\theta d\phi \tag{1.3}$$

The squashing parameter α lies in the range

$$-1 \leq \alpha < \infty \tag{1.4}$$

with $\alpha = 0$ corresponding to the round three sphere. In the large α limit the squashed sphere approaches the direct product space $S^2 \times S^1$, and the periodicity of the S^1 fibre

can be thought of as an inverse temperature. The limit $\alpha \to -1$ is the limit of extreme squashing which was not accessible analytically before, and this will be the main focus of this work. In this limit one of the dimensions becomes very large compared to the others and the field theory becomes effectively one dimensional.

This duality has the advantage compared to the usual string/gauge theory dualities in that the QFT is exactly solvable and can be compared to the semiclassical properties of Einstein's gravity in the absence of a proper formulation of the higher spin gauge theories in AdS Taub-NUT and AdS Taub-Bolt spacetimes. It is useful to solve the $O(N)$ model on a squashed three sphere because it provides a one parameter family of field theory/ gravity dualities, whose free energies exhibit a non monotonic behavior as a function of the squashing parameter as argued in [5]. For other related works on the $O(N)$ model and the Klebanov-Polyakov duality, see [6−8].

The squashed three sphere is the conformal boundary of AdS Taub-NUT and AdS Taub-Bolt geometries [9, 10]. As in the canonical example of the Hawking Page transition [11], only one of these two geometries dominates the partition function. In particular, as a function of α, there is a Hawking-Page transition from AdS Taub-NUT to AdS Taub-Bolt, the latter dominating for large α. In [12, 13] the action of AdS Taub-NUT was found to be:

$$I_{TN} = -\frac{6\pi}{GR}\frac{(1+2\alpha)}{(1+\alpha)^2} \tag{1.5}$$

where G is Newton's constant and R is the Ricci scalar which is negative in these backgrounds. For AdS Taub-Bolt the corresponding result is:

$$I_{TB} = \frac{24\pi}{RG}(1+\alpha)^{-\frac{1}{2}}\left(m_b + \frac{3}{4}r(1+\alpha)^{-1} - r^3\right) \tag{1.6}$$

with

$$m_b = \frac{1}{2}r + \frac{1}{8r}(1+\alpha)^{-1} + \frac{1}{2}\left(r^3 - \frac{3}{2}r(1+\alpha)^{-1} - \frac{3}{16r}(1+\alpha)^{-2}\right) \tag{1.7}$$

and

$$r = \frac{1}{6}(1+\alpha)^{\frac{1}{2}}(1 + (1 - 12(1+\alpha)^{-1} + 9(1+\alpha)^{-2})^{\frac{1}{2}}) \tag{1.8}$$

In the limit of large α the AdS Taub-Bolt action grows linearly:

$$I = \frac{4\pi}{9GR}\alpha \qquad \alpha \to \infty \tag{1.9}$$

The action of the $O(N)$ model was calculated in [5] for $\alpha > -\frac{8}{9}$ and is shown in Fig 1 below, and the result for the gravitational side is shown in Fig 2, where the action has been normalized so that it agrees with the field theory results at large α and a constant has been added so that the peaks coincide. They found a close numerical agreement between the results for the $O(N)$ model and AdS Taub-NUT space below a critical value of α and with AdS Taub-Bolt above it, but with a smooth crossover between the two which suggests that the higher spin gauge fields have the effect of smoothing out the phase transition.

In [5] the large α behavior of the QFT action at strong coupling was found to be:

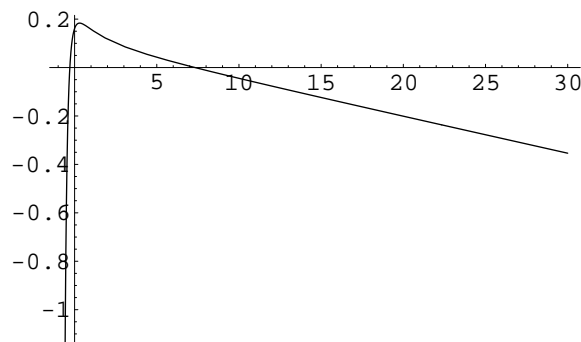$$I_{a\lambda \gg 1} = -\frac{N\zeta_R(3)}{10\pi^2}\alpha \tag{1.10}$$

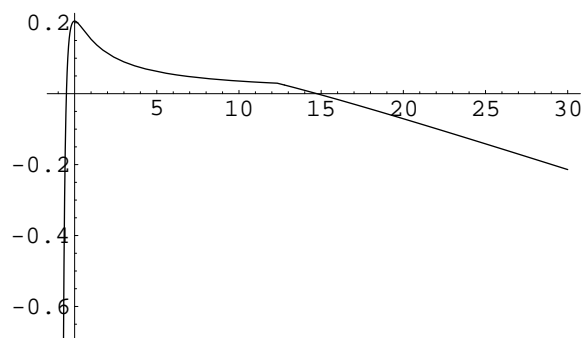**Figure 1:** The action $\frac{I}{N}$ of the $O(N)$ model at strong coupling as a function of α.

Figure 2: The Gravitational action as a function of α.

The linear behaviour is guaranteed by the thermodynamic interpretation which emerges at large α wherein the metric effectively approaches $S^2 \times S^1$.

The striking and somewhat mysterious feature of the agreement found in [5], is that the linear beahviour at large α turns over at small α (see figures 1 and 2). In this note we attempt to understand this non-monotonic behaviour analytically, and in the process go beyond some aspectds of the work of [5] to the limit $\alpha \to -1$. The main result of this paper is the strong coupling action of the $O(N)$ model near the lower limit of the range of the squashing parameter, $\alpha \to -1$:

$$I_{a\lambda \gg 1} = \left(\frac{\ln(1+\alpha)}{3(1+\alpha)^2} + \frac{0.0614093}{(1+\alpha)^2} \right) N \tag{1.11}$$

The results (1.10) and (1.11) are to be compared and contrasted with (1.9) and (1.5).

It can be seen that the qualitative behavior of the free energy of the $O(N)$ model as $\alpha \to -1$ and at large α closely reproduces the results of semiclassical gravity. The logarithmic deviation in the leading order term in the limit $\alpha \to -1$ is a prediction for the effect of including higher spin gauge fields in addition to gravity in the bulk dual. Interestingly there appears to be no a priori reason why the results for the higher spin gauge theory should be so close to the pure gravity result, though these results suggest that the effects of the higher spin gauge fields cannot be drastic.

In section two we summarize some useful results from the $O(N)$ model and in section three we describe the calculation in more detail. Section four contains a discussion and summary.

2. O(N) model in the large N limit

The $O(N)$ model has been extensively studied in various dimensions e.g. see [14]. In Euclidean space the $O(N)$ model has the classical action,

$$S = \int dx^D \sqrt{g} \left(\frac{1}{2} \nabla \Phi \cdot \nabla \Phi + \frac{1}{2} m^2 \Phi \cdot \Phi + \frac{\lambda}{4N} (\Phi \cdot \Phi)^2 \right). \tag{2.1}$$

The coupling constant λ flows from a free fixed point in the UV to another fixed point in the IR. This model can be solved exactly in the strictly large N limit by deriving an effective potential. This can be done by introducing a homogenous background expectation value ϕ for the $O(N)$ field and then splitting the field into a VEV and fluctuations as follows:

$$\Phi = \left(\sqrt{N} \phi + \delta\phi, \pi_1, \pi_2, \ldots \ldots, \pi_{N-1} \right). \tag{2.2}$$

Here ϕ is the homogeneous background and $\delta\phi$ and $\vec{\pi}$ are the fluctuations around it. Normally this would break the $O(N)$ symmetry to $O(N-1)$ resulting in goldstone bosons, however as argued by [5] in these circumstances the symmetry is not broken because the path integral includes an integration over the vacuum manifold which implies that symmetry breaking does not occur in a compact space. The fluctuations can then be integrated out, and the result is:

$$
\begin{aligned}
\frac{V_{\text{eff}}(\phi, \sigma)}{N} =& \frac{1}{2}(m^2 + \lambda\sigma)\phi^2 - \frac{\lambda}{4}\sigma^2 + \frac{1}{2\text{Vol}(M)} \ln \det' \left(\frac{-\Box + m^2 + \lambda\sigma}{\mu^2} \right) \\
& - \frac{1}{2\text{Vol}(M)} \left(1 + \ln \pi + \ln \frac{\phi^2}{\mu} \right),
\end{aligned}
\tag{2.3}
$$

Where $\text{Vol}(M)$ is the volume of the manifold on which the field theory is formulated. In the present context M represents the squashed three sphere. μ is a dimensional scale which is like the sliding renormalization scale. The prime in $\det'$ indicates that the integration was not done over the constant mode which is then dealt with separately. In the large N limit only the configuration obtained by extremising (2.3) contributes to the partition function. Minimizing the effective potential with respect to ϕ and σ yields the equations:

$$\phi^2(m^2 + \lambda\phi) = \frac{1}{\text{Vol}(M)} \tag{2.4}$$

and

$$\phi^2 - \sigma + \frac{1}{\text{Vol}(M)} \text{Tr}' \left(\frac{1}{-\Box + m^2 + \lambda\sigma} \right) = 0. \tag{2.5}$$

An "effective pion mass" can then be defined:

$$m_\pi^2 = m^2 + \lambda\sigma \tag{2.6}$$

so that equations (2.4) and (2.5) can be rewritten as a gap equation for m_π^2

$$m_\pi^2 = m^2 + \frac{\lambda}{\text{Vol}(M)}\text{Tr}\left(\frac{1}{-\Box + m_\pi^2}\right) \qquad (2.7)$$

where the constant mode has been absorbed into the above. Once (2.7) has been solved the effective potential can be evaluated at the extremum to give the action:

$$S = \frac{N}{2}\left(-\frac{\text{Vol}(M)}{2\lambda}(m^2 - m_{\pi^2}^2) + \ln\det\frac{-\Box + m_\pi^2}{\mu^2} + \ln(\mu^3\text{Vol}(M))\right). \qquad (2.8)$$

To evaluate (2.7) it is necessary to evaluate the trace. This can be done by the method of zeta function regularization. The zeta function for an elliptic operator A is defined by

$$\zeta(s) = \text{Tr}A^{-s} \qquad (2.9)$$

so that

$$\ln\det\frac{-\Box + m_\pi^2}{\mu^2} = -\lim_{s\to 0}\frac{d}{ds}\text{Tr}\left(\frac{-\Box + m_\pi^2}{\mu^2}\right)^{-s} = -\zeta'(0). \qquad (2.10)$$

The zeta function on the squashed three sphere can be written in the form [15–17]

$$\zeta(s) = \sum_{l=1}^{\infty}\sum_{q=0}^{l-1}\frac{l(a\mu)^{2s}}{(l^2 + \alpha(l-1-2q)^2 + a^2 m_\pi^2 - 1)^s}, \qquad (2.11)$$

The self-consistent gap equation which determines the solution of the model is highly non-trivial for two reasons. Firstly, it is a a nonlinear equation for m_π^2. Secondly, it involves a zeta function on the squashed sphere, namely $\zeta_{m_\pi^2}(1)$ which is a complicated object and needs to be defined via analytic continuation.

In addition to these ingredients, we need to specify the coupling constant of the theory λ which is dimensionful. The relevant dimensionless parameter in the theory is the combination $a\lambda$. Since λ is a relevant coupling in three dimensions, $a\lambda \ll 1$ is the weak coupling limit corresponding to taking the sphere size to be small, thus approaching the UV free fixed point.

We will be primarily interested in the strong coupling limit $a\lambda \to \infty$ which corresponds to the IR fixed point theory on the squashed sphere. In this limit, a drastic simplification of the gap equation occurs, allowing us to solve the problem analytically in the $\alpha \to -1$ limit. The gap equation at strong coupling (2.7) determines m_π^2 to be a zero of $\zeta_{m_\pi^2}(1)$. The results of [5] provide evidence that the resulting value for m_π^2 is finite and non-negative for all allowed values of the squashing parameter α.

We will now evaluate the action of the theory in the $\alpha \to -1$ limit, at strong coupling $a\lambda \to \infty$. The fact that m_π^2 has a finite value determined by the zero of $\zeta_{m_\pi^2}(1)$, implies that the first term in (2.8) is zero at strong coupling. The volume term will finally be found to give a subleading contribution to the action. The dominant contribution in the $\alpha \to -1$ limit therefore is: $-\frac{N}{2}\zeta'(0)$. As this is superficially divergent it needs to be analytically continued by standard methods described in the appendix.

3. The $O(N)$ model as $\alpha \to -1$

The zeta function (2.11) is superficially divergent, but a finite value may be obtained by analytically continuing the sum, firstly by applying the Abel-Plana formula, and then by carrying out a Sommerfield Watson transformation. The lengthy technical details are summarized in the appendix. We find:

$$\frac{\zeta(s)}{\mu^{2s}} = \frac{a^{2s}A}{(1+\alpha)^s} + a^{2s}\int_0^1 \frac{B(y)dy}{(1+\alpha y^2)^s} - \frac{2ia^{2s}}{(1+\alpha)^s}\int_0^\infty \frac{C(y)dy}{1+\exp(2\pi y)} \tag{3.1}$$

where the functions A, B, C are themselves infinite sums defined in the appendix. After lengthy complex analysis manipulations, using the above result, we find that the zeta function in the limit $\alpha \to -1$, is given by:

$$\begin{aligned}\zeta'(0) = &-\frac{\log(1+\alpha)}{3(1+\alpha)^2} - \frac{10}{9(1+\alpha)^2} - \frac{2\ln 2}{3(1+\alpha)^2} \\ &+ \int_0^\infty \frac{16y\ln(1+4y^2) - 8(4y^2-1)\tan^{-1}2y}{\exp(2\pi y)-1}dy.\end{aligned} \tag{3.2}$$

This then evaluates to:

$$I = \frac{N}{2}\left(\frac{\ln(1+\alpha)}{3(1+\alpha)^2} + \frac{0.0614093}{(1+\alpha)^2}\right). \tag{3.3}$$

The above argument relies on $a^2 m_\pi^2$ being finite at strong coupling and in the entire range of allowed values of α. In fact the numerical results in [5] indicate that $a^2 m_\pi^2$ from the solution to the strong coupling gap equation approaches zero in the $\alpha \to -1$ limit.

4. Summary and discussion

We have solved the strongly coupled $O(N)$ model exactly in the limit $\alpha \to -1$ and found a soft logarithmic deviation from the results of semiclassical gravity in this regime. It is surprising that the scaling of the action of the strongly coupled $O(N)$ model in this limit, is so similar to that of classical gravity on AdS-Taub-NUT space. The "anomalous" logarithmic deviation can only be explained within the confines of the Klebanov-Polyakov conjecture as being due to the effects of the higher spin gauge fields. There appears to be no obvious physical explanation for the behavior of the action in this limit, but it would be interesting to see if it is because in the $\alpha \to -1$ limit the field theory becomes effectively one dimensional.

In the other extreme of the allowed range of α, namely at large α, it is evident from (1.9) and (1.10) that classical gravity and boundary field theory are qualitatively similar. This is not very surprising, since at large α, the boundary theory can be reinterpreted as being at a finite temperature given by α. The linear scaling of the action with α, and equivalently the free energy scaling as α^2, is what one expects in a field theory in three dimensions. Nevertheless, from this we learn, assuming the validity of the Klebanov-Polyakov conjecture, that the higher spin theory dual to the $O(N)$ model at large squashing, should behave

in qualitatively the same fashion as Einstein gravity in AdS-Taub-Bolt space. We remark that the coefficients for the field theory (1.10) and gravity (1.9) actions are not expected to match as the higher spin gauge fields were not included in the gravity calculation. In any case, matching of these coefficients only sets up the dictionary between $1/N$ in field theory and the bulk curvature in units of the 4d Planck mass. The above discussion compares with the AdS_5/CFT_4 case where doing a strongly coupled field theory calculation is difficult and there is a 3/4 discrepancy factor between strong and weak 't Hooft coupling results due to higher stringy modes becoming light at large, string-scale curvatures in the string dual of the weakly coupled gauge theory.

The analytic results obtained in this paper, for the strongly coupled field theory near $\alpha \to -1$, when combined with the linear behaviour at large α, reproduce remarkably well the non-monotonic behaviour of the classical bulk gravity action presented in figure 2. Note that the non-monotonic behaviour in the bulk (without higher spin fields) is due to a Hawking-Page transition which is necessary in order to pass over from the AdS-Taub-NUT to the AdS-Taub-Bolt phase, the latter showing a linear beahviour with α at large α. The message is that even though we don't have a proper formulation of the higher spin theory in these backgrounds, our results suggest that gravity reproduces qualitatively similar results to the higher spin gauge theory dual to the $O(N)$ model.

Finally, AdS Taub Nut space is obtained by filling the volume of a squashed three sphere with a hyperbolic metric with negative cosmological constant [18]. In the limit $\alpha \to -1$ the space becomes a Bergmann space which can be described as a coset space $SU(2,1)/U(2)$ which has been studied in [19]. It would be interesting to understand the behavior of the action from the bulk perspective by considering higher spin gauge fields on this Bergmann space. For a detailed construction of bulk-boundary and bulk-bulk propagators in this space see [20]. Other work in AdS Taub Nut space is contained in [21 – 23].

Acknowledgments

A. Analytic continuation of the zeta function

As (2.11) is divergent if s is set directly to zero it will need to be continued analytically. This was done by converting the sum over q into an integral using the Abel-Plana formula and evaluating the l summation using a Sommerfield-Watson transformation. (2.11) has branch cuts at

$$q = \frac{l-1}{2} \pm \frac{1}{2}\left(\frac{1-l^2-a^2m_\pi^2}{\alpha}\right)^{1/2}, \tag{A.1}$$

with these branch cuts the Abel-Plana formula of the form (A.2) may be used to evaluate the l sum.

$$\sum_{i=n}^{m} \phi(x) = \frac{1}{2}(\phi(n) + \phi(m)) + \int_{n}^{m} \phi(x)dx$$
$$- i \int_{0}^{\infty} \frac{dy}{\exp(2\pi y) - 1}(\phi(n - iy) - \phi(n + iy) - \phi(m - iy) + \phi(m + iy)). \tag{A.2}$$

Using this we obtain:

$$\frac{\zeta(s)}{\mu^{2s}} = \frac{a^{2s}A}{(1+\alpha)^s} + a^{2s} \int_{0}^{1} \frac{B(y)dy}{(1+\alpha y^2)^s} - \frac{2ia^{2s}}{(1+\alpha)^s} \int_{0}^{\infty} \frac{C(y)dy}{\exp(2\pi y) - 1}, \tag{A.3}$$

where:

$$A = \sum_{l=1}^{\infty} \frac{l}{((l+G)^2 - h^2)^s}, \tag{A.4}$$

$$B = \sum_{l=1}^{\infty} \frac{l(l-1)}{((l+I)^2 - J^2)^s}, \tag{A.5}$$

$$C = \sum_{l=1}^{\infty} l\frac{1}{((l+K)^2 - M^2)^s} - \frac{1}{((l+K^*)^2 - (M^2)^*)^s}. \tag{A.6}$$

Here G, H, I, J, K, M are given by:

$$G = \frac{-\alpha}{1+\alpha} \tag{A.7}$$

$$-H^2 = \frac{a^2(m_\pi)^2(1+\alpha) - 1}{(1+\alpha y^2)^2}, \tag{A.8}$$

$$I = \frac{-\alpha y^2}{(1+\alpha y^2)}, \tag{A.9}$$

$$-J^2 = \frac{a^2 m_\pi^2 (1+\alpha y^2) - 1}{(1+\alpha y^2)^2}, \tag{A.10}$$

$$K = \frac{\alpha(-1+2iy)}{1+\alpha}, \tag{A.11}$$

$$-M^2 = \frac{a^2 m_\pi^2 - 1 + \alpha(-1+2iy)}{1+\alpha} - \frac{\alpha^2(-1+2iy)^2}{(1+\alpha)^2}, \tag{A.12}$$

The sums over l can be evaluated using a Sommerfield-Watson transformation. Apply this to the A we find

$$A = \frac{i}{2} \int_{C_1} \frac{z \cot \pi z dz}{((z+G)^2 - H^2)^s}, \tag{A.13}$$

where the contour C_1 is shown in figure three. For $\text{Re}(s) > 2$ this can be deformed into C_2, also shown in figure three.

It is useful to rewrite $\cot \pi z$ with the identities

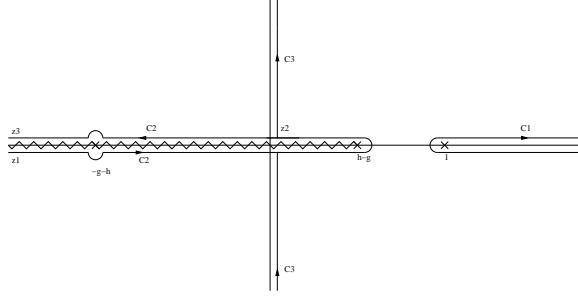$$\cot \pi z = i\left(1 + \frac{2}{\exp 2i\pi z - 1}\right), \tag{A.14}$$

Figure 3: The contours used for the analytic continuation of the zeta function.

between z_1 and z_2 and

$$\cot \pi z = i\left(-1 + \frac{2}{1 - \exp(2i\pi z)} \right), \tag{A.15}$$

between z_2 and z_3.

The integrals over the exponential pieces are then manifestly finite and can be evaluated along C_3. The integrals over the constant pieces can be done analytically for $\text{Re}(s) > 2$. These expressions are then evaluated at $s = 0$ to define the analytically continued function. A similar method is used for B and C. In the case of C the branch points are not on the real axis so the integrals are no longer along the real axis but along a tilted contour. Using this method the following results are obtained in the limit α tends to minus one.:

$$A \mid_{s=0} = \frac{1}{(1+\alpha)^2}, \tag{A.16}$$

$$\frac{d}{ds} A \mid_{s=0} = \frac{2\log(1+\alpha)}{(1+\alpha)^2} + \frac{3}{(1+\alpha)^2} - \frac{2\log 2}{(1+\alpha)^2}, \tag{A.17}$$

$$\int_0^1 B \mid_{s=0} (2\log a - \log(1+\alpha y^2)) = -\frac{4\log a}{3(1+\alpha)^2} + \frac{1}{3(1+\alpha)^2} + \frac{2\log(1+\alpha)}{3(1+\alpha)^2}, \tag{A.18}$$

$$\int_0^1 \frac{dB}{ds} \mid_{s=0} = \frac{4\log(1+\alpha)}{3(1+\alpha)^2} - \frac{10}{9(1+\alpha)^2} - \frac{4\log 2}{3(1+\alpha)^2} \tag{A.19}$$

$$C \mid_{s=0} = -\frac{8yi}{(1+\alpha)^2}, \tag{A.20}$$

$$\frac{dC}{ds} \mid_{s=0} = \frac{2i}{(1+\alpha)^2}(4y(-7+8\log 2 - 4\log(1+\alpha) + 2\log(1+4y^2))$$
$$-4(4y^2 - 1)\tan^{-1} 2y). \tag{A.21}$$

Putting the above together gives:

$$\zeta'(0) = -\frac{\log(1+\alpha)}{3(1+\alpha)^2} - \frac{10}{9(1+\alpha)^2} - \frac{2\ln 2}{3(1+\alpha)^2}$$
$$+ \int_0^\infty \frac{16y\ln(1+4y^2) - 8(4y^2 - 1)\tan^{-1} 2y}{\exp(2\pi y) - 1} dy. \tag{A.22}$$

The integrals can then be evaluated numerically.

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